I. Solution by the PROPOSER.

It can be demonstrated that D, in $x^2-Ay=\pm D$, can be any denominator of the complete quotients from the $\sqrt{A}$, and that x and y are the numerator and denominator of the convergent preceding the term in which D is taken. Now the complete quotients for the $\sqrt{114\frac{1}{4}}$ are

$$\frac{0+\sqrt{114\frac{1}{4}}}{1};\quad \frac{10\frac{1}{2}+\sqrt{114\frac{1}{4}}}{4};\quad \frac{9\frac{1}{2}+\sqrt{114\frac{1}{4}}}{6};\quad \frac{8\frac{1}{2}+\sqrt{114\frac{1}{4}}}{7};\ \text{etc.}$$

No. term......	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	etc., reversing.
$\therefore \sqrt{114\frac{1}{4}}=\frac{1}{0}, \frac{10\frac{1}{2}}{1}$:	5,	3,	2,	1,	2,	1,	6,	2,	1,	1,	10,	10,	etc., reversing.
Complete denom'rs=1 :	4,	6,	7,	12,	6,	14,	3,	8,	9,	12,	2,	2,	etc., reversing.

Hence x and y are found in the 6th convergent and also the 15th convergent, and x and $y=2095$ and 196, and also from the 15th term $x=42,307,834$, and $y=3,958,154$. [Also see problem 38.]

II. Solution by JOSIAH H. DRUMMOND, LL. D., Portland Maine.

I have not solved this problem as stated, but as I have solved it in this form $x^2-114\frac{1}{4}y^2\pm3=\square$ ......(1), and as that is a pretty question, I send my solution.

Multiplying by 4, it becomes $4x^2-457y^2\pm12=\square=$say $(2x-m^2=4x^2-4mx+m^2)$, from which we find

$$x=\frac{457y^2+m^2\pm12}{4m}.$$

$(m\pm12)/4m$ evidently becomes integral when $m=6$; and we have

$$x=\frac{457y^2}{24}+2,\ \text{or}\ \frac{457y^2}{24}+1.$$

$457y^2/24$ becomes integral when $y=12n$, and $x=2742n+2$, or $=2743n+1$, according as the $+$ or $-$ sign before 3 is taken.

If $n=1$, $y=12$, and $x=2744$ or 2743 ; in the former, 3 is negative, and in the latter, positive, in order to make the expression a square.

54. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

In the expression $2x^2-2ax+b^2$, find two series of values for x in integral terms of a and b.

I. Solution by the PROPOSER.

$2x^2-2ax+b^2$ is evidently a square when $x=a$. Take $x=y+a$, and substituting, we have $2y^2+2ay+b^2=\square=$(say)$(my-b)^2$.

Reducing, $y=2(a+bm)/(m^2-2)$. Taking $m=2/1$, $10/7$, $58/41$, etc., we have *one* integral series of the value of y, viz : $a+2b$, $49a+10b$, etc. Taking $m=3/2$, $17/12$, $99/70$, etc., we have *another* integral series of the value of y, viz :

$8a+12b$, $288a+408b$, etc. By adding a to each term of each series we have two series of the value of x. These series hold good when either a or b is zero ; but if both are zero, $x=0$.

It will be noticed that this solution applies the terms of the question to the expression $2x^2+2ax+b=\square$, the value of x in the latter being a less than in the former.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.**

$2x^2-2ax+b^2=0$. $\therefore x=\frac{1}{2}(a\pm\sqrt{a^2-2b^2})$.

Let $a=p^2+2q^2$, $b=2pq$. Then $x=p^2$ or $2q^2$.

$\therefore$ p	q	a	b	x,
2	1	6	4	4 or 2,
3	2	17	12	9 or 8,
4	3	34	24	16 or 18,
1	2	9	4	1 or 8,
etc.	etc.	etc.	etc.	etc.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

54. **Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.**

A man is at the center of a circle whose diameter is equal to three of his steps. If each step is taken in a perfectly random direction, what is the probability, (1), that he will step outside the circle at the second step, and, (2), that he will step outside at the third step ?

I. **Solution by the PROPOSER.**

Let O be the center of the circle, A, the end of the first step, and B, the end of the second, and C, the end of the third.

Let $\angle OAB=\theta$, $\angle OBC=\phi$, $OB=x$, and $OC=y$.

Then if the length of the step be taken as the unit of measure, $x=2\sin\frac{1}{2}\theta$, and $y=(x^2+1-2x\cos\phi)^{\frac{1}{2}}=(4\sin^2\frac{1}{2}\theta+1-4\sin\frac{1}{2}\theta\cos\phi)^{\frac{1}{2}}$.

If $x=\frac{3}{2}$, B falls upon the circumference of the circle, and $\theta=2\sin^{-1}\frac{3}{4}$. If θ be $>2\sin^{-1}\frac{3}{4}$, and $<\pi$, the second step falls outside the circle. The probability of this is $P_1=(\pi-2\sin^{-1}\frac{3}{4})/\pi$.

If θ be $<2\sin^{-1}\frac{3}{4}$, and $y=\frac{3}{2}$, C falls upon the circumference of the circle, and $4\sin^2\frac{1}{2}\theta+1-4\sin\frac{1}{2}\theta\cos\phi=\frac{9}{4}$ or $\phi_1=\cos^{-1}(\sin\frac{1}{2}\theta-5/16\sin\frac{1}{2}\theta)$. Hence if ϕ be $>\phi_1$ the third step falls outside the circle. The chance that ϕ will be $>\phi_1$ and $<\pi$ is $(\pi-\phi_1)\pi$. The chance that θ has any particular value is $d\theta/\pi$. Hence the probability that the third step falls outside the circle is